Design of Digital Differentiators Using Interpolation and Model Order Reduction Technique

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Abstract – In this paper, first order digital differentiators are obtained by the combination of interpolation and model order reduction techniques. MATLAB simulation results have been presented to validate the effectiveness of the proposed differentiators. The designed differentiators have shown superior performance compares to Al-Alaoui and Ngo differentiators at low frequency region.

Index Terms – Digital differentiator, Interpolation, Model order reduction, Discretization, Al-Alaoui operator.

1. INTRODUCTION

Digital differentiators are widely used in many areas of engineering like Radar, Sonar, Image Processing, Signal Processing, Bio-Medical Engineering, etc.[1-7]. Infinite Impulse Response (IIR) type digital differentiators are suitable for real time applications compared to the Finite Impulse response (FIR) type. Various approximations of ideal differentiators have been stated in the literature [1-7].AI-Alaoui has designed a first order digital differentiator based on the interpolation of Trapezoidal and Rectangular Integration formulae [1-2]. In general IIR type digital differentiators are designed based on discretization of Newton-Cotes Integration rule [3].Ngo [3] used the closed form of the Newton-Cotes integration rule to design a digital integrator. A class of digital integrators and differentiators has also been proposed by Al-Alaoui [4] by using interpolation and a simulated annealing optimization method. Gupta-Jain-Kumar (GJK) [5] have also suggested digital integrators by using the interpolation method. In [6]

G. Visweswaran et.al have designed new type of digital differentiators using a class of model reduction technique which have shown greater performance in the low-frequency range. In this paper, novel digital differentiators are proposed by using both Interpolation technique in [2] and Model order reduction technique proposed in [6].

The organization of this paper is as follows: Section 2 presents the digital integrator and their comparison. Section 3 presents design of digital differentiator using Interpolation technique and Model order reduction technique. All the figures in this paper were obtained by using MATLAB. Section 4 concludes the paper.

2. DIGITAL INTEGRATORS

Backward or rectangular integrator is defined as [1]

$$H_1(z) = \frac{zT}{z-1} \quad (1)$$

Where T is sampling time. Tustin or trapezoidal integrator is defined as [1]

$$H_2(z) = \frac{T(z+1)}{2(z-1)} \quad (2)$$

By interpolating these two integrators, Ai-Alaoui obtained a new integrator $H_3(z)$

$$H_3(z) = \frac{T(z+7)}{8(z-1)} \quad (3)$$

Simpson's integrator is defined as [2]

$$H_4(z) = \frac{T(z^2+4z+1)}{3(z^2-1)} \quad (4)$$

Comparison of all integrators is shown in Figure 1. From the figure it can be observed that, the magnitude response of an ideal integrator lies between 1) Trapezoidal and Simpson integrators 2) Rectangular and Al-Alaoui integrators 3) Al

Alaoui and Simpson integrators 4) Rectangular and Trapezoidal integrators.

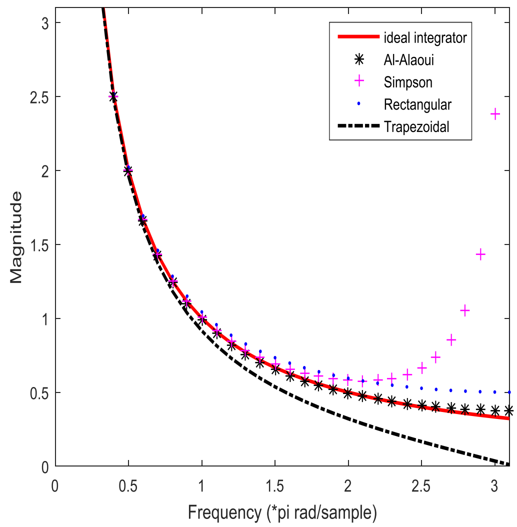

Figure 1 Comparison of magnitude responses of IIR type digital integrator

3. DESIGN OF DIGITAL DIFFERENTIATOR USING INTERPOLATION TECHNIQUE AND MODEL ORDER REDUCTION TECHNIQUE

3.1. INTERPOLATED TRAPEZOIDAL AND SIMPSON DIFFERENTIATOR

By interpolating $H_2(z)$ and $H_4(z)$

$$H_{N1}(z) = \alpha H_2(z) + (1-\alpha)H_4(z) \qquad (5)$$

Where $0 < \alpha < 1$. Substituting $H_2(z)$ and $H_4(z)$ and simplifying

$$H_{N1}(z) = \alpha\left\{\frac{T(z+1)}{2(z-1)}\right\} + (1-\alpha)\left\{\frac{T(z^2+4z+1)}{3(z^2-1)}\right\} \qquad (6)$$

$$H_{N1}(z) = \frac{(2+\alpha)T}{6(z^2-1)}\left[z^2 + \frac{(8-2\alpha)}{(2+\alpha)}z + 1\right] \qquad (7)$$

Substituting $z = 1 - \delta z$ in Eqn. (7) and neglecting δz^2 term, $H(z)$ reduces to

$$H_{N1}(z) = \frac{T(1-\delta z)}{-\delta z} \qquad (8)$$

Replacing $-\delta z = z - 1$, and simplifying

$$H_{N1}(z) = \frac{Tz}{(z-1)} \qquad (9)$$

Inverting Eqn. (9)

$$G_{N1}(z) = \frac{(z-1)}{Tz} \qquad (10)$$

Eqn. (10) represents Backward or Rectangular differentiator.

3.2. INTERPOLATED BACKWARD AND AL-ALAOUI DIFFERENTIATOR

By interpolating $H_1(z)$ and $H_3(z)$ [7]

$$H_N(z) = \alpha H_1(z) + (1-\alpha)H_3(z) \qquad (11)$$

Where $0 < \alpha < 1$. Substituting $H_1(z)$ and $H_3(z)$ and simplifying

$$H_N(z) = \frac{T[(7\alpha+1)z + 7(1-\alpha)]}{8(z-1)} \qquad (12)$$

By inverting the above equation, the new differentiator, $G_N(z)$ will be

$$G_N(z) = \frac{8(z-1)}{T[(7\alpha+1)z + 7(1-\alpha)]} \qquad (13)$$

Using pole stabilization

$$G_N(z) = \frac{8(z-1)}{7T(1-\alpha)[z + \frac{(7\alpha+1)}{7(1-\alpha)}]} \qquad (14)$$

For the above differentiator to be stable [2],

$$\frac{7\alpha+1}{7(1-\alpha)} \le 1 \implies \alpha \le \frac{3}{7} \qquad (15)$$

α	$G_N(z)$
0	$\frac{8(z-1)}{7T(z+\frac{1}{7})}$
$\frac{0.1}{7}$	$\frac{8(z-1)}{6.9T(z+\frac{11}{69})}$
$\frac{0.2}{7}$	$\frac{8(z-1)}{6.8T(z+\frac{12}{68})}$
$\frac{0.3}{7}$	$\frac{8(z-1)}{6.8T(z+\frac{13}{67})}$
$\frac{1}{7}$	$\frac{8(z-1)}{6T(z+\frac{1}{3})}$
$\frac{2}{7}$	$\frac{8(z-1)}{5T(z+\frac{3}{5})}$
$\frac{3}{7}$	$\frac{2(z-1)}{T(z+1)}$

Table 1 Novel differentiator for different values of α

3.3. INTERPOLATED AL-ALAOUI AND SIMPSON'S DIFFERENTIATOR

By interpolating $H_3(z)$ and $H_4(z)$

$$H(z) = \alpha\left\{\frac{T(z+7)}{8(z-1)}\right\} + (1-\alpha)\left\{\frac{T(z^2+4z+1)}{3(z^2-1)}\right\} \quad (16)$$

Where $0 < \alpha < 1$. Simplifying Eq. (16)

$$H(z) = \frac{(8-5\alpha)T\left\{z^2+\frac{8(4-\alpha)}{8-5\alpha}z+\frac{8+13\alpha}{8-5\alpha}\right\}}{24(z^2-1)} \quad (17)$$

Substituting $z = 1 - \delta z$ in Eqn. (17) and neglecting δz^2 term, $H(z)$ reduces to

$$H(z) = \frac{T(8-5\alpha)\left[-\delta z\left\{\frac{48-18\alpha}{8-5\alpha}\right\}+\frac{48}{8-5\alpha}\right]}{-48\delta z} \quad (18)$$

Replacing $-\delta z = z - 1$, and simplifying

$$H(z) = \frac{T(24-9\alpha)\left[z+\frac{9\alpha}{24-9\alpha}\right]}{24(z-1)} \quad (19)$$

Inverting Eqn. (19) and stabilizing

$$G(z) = \frac{8}{(8-3\alpha)T}\frac{z-1}{z+\frac{3\alpha}{8-3\alpha}} \quad (20)$$

For the above differentiator to be stable,

$$\alpha \leq \frac{4}{3} \quad (21)$$

α	$G(z)$
0	$\frac{z-1}{T(z)}$
0.25	$\frac{1.1032(z-1)}{T(z+0.1988)}$
0.275	$\frac{1.115(z-1)}{T(z+0.115)}$
$\frac{1}{3}$	$\frac{8(z-1)}{7T(z+\frac{1}{7})}$
0.35	$\frac{1.151(z-1)}{T(z+0.151)}$
0.425	$\frac{1.19(z-1)}{T(z+0.19)}$
0.5	$\frac{1.2308(z-1)}{T(z+0.23077)}$
1	$\frac{8(z-1)}{5T(z+\frac{3}{5})}$
1.25	$\frac{1.8824(z-1)}{T(z+0.8824)}$
$\frac{4}{3}$	$\frac{2(z-1)}{T(z+1)}$

Table 2 Novel differentiator for different values of α

The magnitude response, phase response and error curve of the above differentiators is shown in Figs. 2,3,4,5 and 6 respectively.

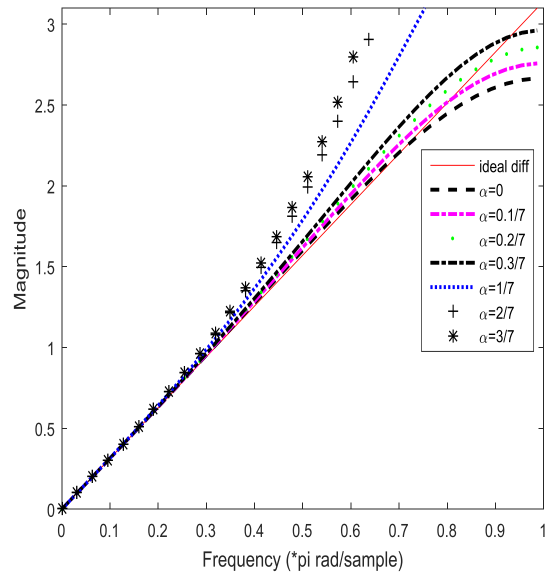

Figure 2 Magnitude responses of Novel differentiators $G_N(z)$ for different values of α

4. CONCLUSION

Novel digital differentiators are presented. By the interpolation of Trapezoidal and Simpson's integrators and applying model reduction technique the Rectangular differentiator obtained. By the interpolation of Backward and Simpson's integrators the differentiators obtained for $\alpha \leq \frac{0.1}{7}$ etc. have shown less error. By the interpolation of Al-Alaoui and Simpson's integrators and using model reduction technique, for $\alpha < 0.3$, the low frequency error is reduced compared to Al-Alaoui and Ngo differentiators. The designed differentiators have shown superior performance compares to Al-Alaoui and Ngo differentiators at low frequency region. As the value of α is increased there is an increment in error but frequency range of operation is enhanced. So these first order differentiators can be used as wide band differentiators.

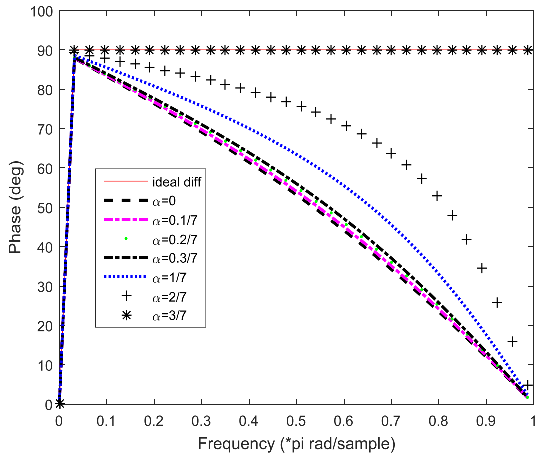

Figure 3 Phase responses of Novel differentiators $G_N(z)$ for different values of α

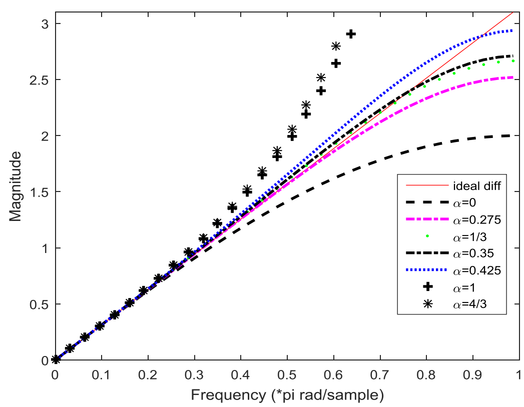

Figure 4 Magnitude responses of Novel differentiators $G(z)$ for different values of α

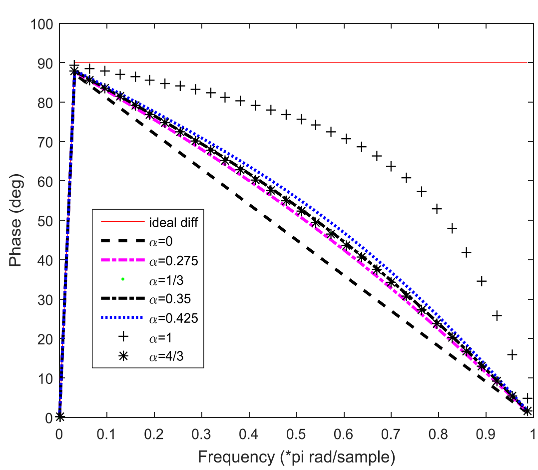

Figure 5 Phase responses of Novel differentiators $G(z)$ for different values of α

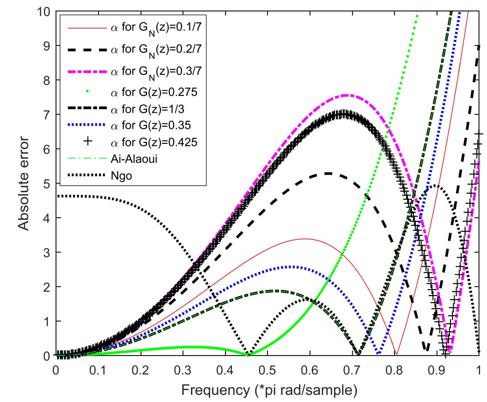

Figure 6 Error curves of differentiators

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